

# On the Relation between the True Directions of Neutrinos and the Reconstructed Directions of Neutrinos in L/E Analysis Performed by Super-Kamiokande Collaboration Part 1

— The Effect of Quasi-Elastic Scattering over the Directions of the Emitted Leptons with regard to Neutrinos among Fully Contained Events —

E. Konishi<sup>a,\*</sup>, Y. Minorikawa<sup>b</sup>, V.I. Galkin<sup>c</sup>, M. Ishiwata<sup>d</sup>, I. Nakamura<sup>e</sup>, N. Takahashi<sup>a</sup>, M. Kato<sup>f</sup>, A. Misaki<sup>g</sup>

<sup>a</sup> Graduate School of Science and Technology, Hirosaki University, Hirosaki, 036-8561, Japan

<sup>b</sup> Department of Science, School of Science and Engineering, Kinki University, Higashi-Osaka, 577-8502, Japan

<sup>c</sup> Department of Physics, Moscow State University, Moscow, 119992, Russia

<sup>d</sup> Department of Physics, Faculty of Science and Technology, Meisei University, Tokyo, 191-8506, Japan

<sup>e</sup> Comprehensive Analysis Center for Science, Saitama University, Saitama, 338-8570, Japan

<sup>f</sup> Kyowa Interface Science Co., Ltd., Saitama, 351-0033, Japan

<sup>g</sup> Innovative Research Organization, Saitama University, Saitama, 338-8570, Japan

---

## Abstract

It should be regarded that the confirmation of the maximum oscillation in neutrino oscillation through  $L/E$  analysis by Super-Kamiokande is a logical consequence of their establishment on the existence of neutrino oscillation through the analysis of the zenith angle distribution for atmospheric neutrino events. In the present paper (Part 1) with the computer numerical experiment, we examine the assumption made by Super-Kamiokande Collaboration that the direction of the incident neutrino is approximately the same as that of the produced lepton, which is the cornerstone in their  $L/E$  analysis, and we find this approximation does not hold even approximately. In a subsequent paper (Part 2), we apply the results from Figures 16, 17, 18 and 19 to  $L/E$  analysis and conclude that one cannot obtain the maximum oscillation in  $L/E$  analysis in the single ring muon events due to quasi-elastic scattering reported by Super-Kamiokande which shows strongly the oscillation pattern from the neutrino oscillation.

PACS: 13.15.+g, 14.60.-z

**Keywords:** Super-Kamiokande Experiment, QEL, Numerical Computer Experiment, Neutrino Oscillation, Atmospheric neutrino

---

## 1. Introduction

<sup>1</sup> In principle, it is pretty difficult to specify the oscillation parameters in neutrino oscillation reliably from the cosmic ray experiments (atmospheric

neutrinos), even if they really exist, because in the cosmic ray experiment by its nature, one cannot determine the direction of the incident neutrino which plays a decisive role in the analysis of neutrino oscillation and may be determined in the accelerator experiments even if the experimental errors are so large due to their geometries.

On the other hand, according to the results obtained from the Super-Kamiokande Experiments on atmospheric neutrinos, it is said that oscillation phenomena have been found between two neutrinos,  $\nu_\mu$  and  $\nu_\tau$ . Published reports on the confirmation to the oscillation between the neutrinos,  $\nu_\mu$  and  $\nu_\tau$ ,

---

\*Corresponding author

Email address: konish@si.hirosaki-u.ac.jp

(E. Konishi)

<sup>1</sup> In order to understand the text of our paper well, we strongly suggest readers to look at the same paper on the WEB where every figures are presented in colors, because figures with colors are strongly impressive compare with those with monochrome. In the figures with colors, we classify neutrino events by blue points and anti-neutrino events by orange ones.

and the history foregoing these experiments will be critically reviewed and details are in the following:

- (1) During 1980's Kamiokande and IMB observed the smaller atmospheric neutrino flux ratio  $\nu_\mu/\nu_e$  than the predicted value [1].
- (2) Kamiokande found anomaly in the zenith angle distribution [2].
- (3) Super-Kamiokande found  $\nu_\mu$ - $\nu_\tau$  oscillation [3] and Soudan2 and MACRO confirmed the Super-Kamiokande result [4].
- (4) K2K, the first accelerator-based long baseline experiment, confirmed atmospheric neutrino oscillation[5].
- (5) MINOS's precision measurement gives the consistent results with Super-Kamiokande ones[6].

It is well known that Super-Kamiokande Collaboration examined all possible types of the neutrino events, such as, say, Sub-GeV e-like, Multi-GeV e-like, Sub-GeV  $\mu$ -like, Multi-GeV  $\mu$ -like, Multi-ring Sub-GeV  $\mu$ -like, Multi-ring Multi-GeV  $\mu$ -like, *Upward Stopping Muon Events* and *Upward Through Going Muon Events*. In other words, all possible interactions by neutrinos, such as, elastic and quasielastic scatterings, single-meson production and deep scattering are considered in their analyses. As the results of them, all topologically different types of neutrino events lead to the unified numerical oscillation parameters, say,  $\Delta m^2 = 2.4 \times 10^{-3} \text{eV}^2$  and  $\sin^2 2\theta = 1.0$  [7].

Really, these parameters are obtained from the unified analysis of the zenith angle distributions of various neutrino events. AS for reliability of the energy estimation which plays a decisively important role in the survival probability of a given flavor, the qualities of the events range in various grades from the coarse to the fine. Generally, it is impossible to estimate the individual energy of the muon due to the neutrino interaction among *Partially Contained Events* due to their stochastic characters. In the multi-ring events, energy estimations has much uncertainty, even if they belong to *Fully Contained Events*. Moreover, it is absolutely impossible to estimate energy of the individual event in *Upward Going Muon Events*.

Consequently, in spite of such a fatal defect for the detection of the neutrino oscillation, which is inherent in the cosmic ray experiments as mentioned above, if one goes ahead and challenges to

prove the existence of the neutrino oscillation in cosmic ray experiments, then, one should focus on analysis of the most reliable events with the highest quality exclusively from the experimental point of view, discarding all other physical events which have uncertainties more or less compared with the events with the highest quality. Such the most reliable events among all candidates to be analyzed for Super-Kamiokande Collaboration are *Fully Contained Events* among the single ring muon events due to the quasi elastic scattering (QEL). These events occupy the majority among *Fully Contained events* observed by Super-Kamiokande Collaboration. By the definition of Fully Contained Events of the single ring events due to QEL, one can discriminate kinds of neutrinos and can estimate their transferred energies because of their confinement in the detector as well as their zenith angles which are strongly connected with their emitted angles in the neutrino interactions concerned. If one can find some clear indication on the neutrino oscillation from the analysis of the most reliable events with the highest quality, then, one can surely expect any corroboration on the neutrino oscillation from the analysis of neutrino events with poorer qualities, if they really exist. This is our motivation for the performance of our computer numerical experiment.

Here, it should be emphasized strongly that Super-Kamiokande Collaboration put a fundamental assumption through their whole analyses. This assumption, however, is never self-evident and, therefore, it should be carefully examined. This assumption is that the directions of the incident neutrinos are approximately the same as those of emitted leptons, which we abbreviate as *the SK assumption on the direction* in the present paper. This assumption should be recognized as the cornerstone for their neutrino oscillation analysis throughout all their works, which links with the survival probability of a given flavor,  $P(\nu_\mu \rightarrow \nu_\mu)$  given by

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \cdot \sin^2(1.27 \Delta m^2 L_\nu / E_\nu), \quad (1)$$

where  $\sin^2 2\theta = 1.0$  and  $\Delta m^2 = 2.4 \times 10^{-3} \text{eV}^2$  [7].

The most important result among the achievements on neutrino oscillation made by Super-Kamiokande Collaboration is the finding of the

maximum oscillation in neutrino oscillation, because it is the ultimate result in the sense that they observe the oscillation pattern itself directly in neutrino oscillation.

Therefore, it is desirable to carry out careful  $L/E$  analysis for the single ring muon events due to QEL (the events with the highest quality) among *Fully Contained Events* exclusively in order to obtain a definite conclusion around the neutrino oscillation, focusing on the validity (or invalidity) of *the SK assumption on the direction*. Consequently, let us examine *the SK assumption on the direction*. This is the main theme of the present paper (Part 1). In a subsequent paper (Part 2), we discuss the application of the result obtained in Part 1 to the analysis for the single ring muon events due to QEL among *Fully Contained Events*, concluding the invalidity of *the SK assumption on the direction* through  $L/E$  analysis.

In order to avoid any misunderstanding toward *the SK assumption on the direction*, we reproduce this assumption from the original SK papers and their related papers in italic.

[A] Kajita and Totsuka [8] state <sup>2</sup>:

*"However, the direction of the neutrino must be estimated from the reconstructed direction of the products of the neutrino interaction. In water Cherenkov detectors, the direction of an observed lepton is assumed to be the direction of the neutrino. Fig.11 and Fig.12 show the estimated correlation angle between neutrinos and leptons as a function of lepton momentum. At energies below 400 MeV/c, the lepton direction has little correlation with the neutrino direction. The correlation angle becomes smaller with increasing lepton momentum. Therefore, the zenith angle dependence of the flux as a consequence of neutrino oscillation is largely washed out below 400 MeV/c lepton momentum. With increasing momentum, the effect can be seen more clearly."*

[B] Ishitsuka [9] states<sup>3</sup>:

*" 8.4 Reconstruction of  $L_\nu$*

*Flight length of neutrino is determined from the neutrino incident zenith angle,*

*although the energy and the flavor are also involved. First, the direction of neutrino is estimated for each sample by a different way. Then, the neutrino flight length is calculated from the zenith angle of the reconstructed direction.*

#### 8.4.1 Reconstruction of Neutrino Direction

##### FC Single-ring Sample

*The direction of neutrino for FC single-ring sample is simply assumed to be the same as the reconstructed direction of muon. Zenith angle of neutrino is reconstructed as follows:*

$$\cos \Theta_\nu^{rec} = \cos \Theta_\mu \quad (8.17)$$

*,where  $\cos \Theta_\nu^{rec}$  and  $\cos \Theta_\mu$  are cosine of the reconstructed zenith angle of neutrino and muon, respectively."*

[C] Jung, Kajita *et al.* [10] state <sup>4</sup>:

*"At neutrino energies of more than a few hundred MeV, the direction of the reconstructed lepton approximately represents the direction of the original neutrino. Hence, for detectors near the surface of the Earth, the neutrino flight distance is a function of the zenith direction of the lepton. Any effects, such as neutrino oscillations, that are a function of the neutrino flight distance will be manifest in the lepton zenith angle distributions."*

As clarified from Figures 11, 12 and 26 in [8] and Figure 8.12 in [9], Super-Kamiokande Collaboration know well the existence of non-negligible scattering angles in the neutrino interactions. Nevertheless, the reason why they put the fundamental postulate on the direction mentioned above is surmised to be due to their recognition that statistically enough accumulation of the events leads to the  $\cos \theta_\nu = \cos \theta_\mu$  as a whole after mutual cancellation of the scattered angles.

As already stated, among all possible events to be analyzed the most important events from which we

<sup>2</sup>see page 101 of the paper concerned.

<sup>3</sup>see page 138 of the paper concerned.

<sup>4</sup>see page 453 of the paper concerned.

can extract neutrino oscillation parameters definitely are undoubtedly the single ring muon (electron) events which are generated in the detector and terminate in it, because these events give the essential information for clear interpretation on neutrino oscillation, if it really exists, namely, the kinds of the neutrinos, the transferred energies to the charged particles and their directions. The single muon events are generated mainly from the quasi-elastic scattering(QEL). If the existence of neutrino oscillation is verified definitely in the analysis of single ring muon events among *Fully Contained Events* under *the SK assumption on the direction*, then again we can say, we expect the corroborations even in the analysis of other types of the interactions with poorer qualities. Therefore, the analysis for the single ring muon events among *Fully Contained Events* is decisively important compared with the analysis of other types of the neutrino events. Consequently, first of all, let us start to examine the validity on *the SK assumption on the direction*.

Our paper is organized as follows. In section 2, we treat the differential cross section for QEL in the stochastic manner as exactly as possible and obtain the zenith angle distributions of the emitted leptons,  $\cos\theta_\mu$ , for given neutrinos with definite zenith angles, taking account of the azimuthal angles of the emitted leptons in QEL. As a result of it, we show that *the SK assumption on the direction* does not hold any more for the incident neutrinos with smaller energies.

In section 3, we give the correlation between  $\cos\theta_{\nu(\bar{\nu})}$  and  $\cos\theta_{\mu(\bar{\mu})}$  or the correlation between  $L_{\nu(\bar{\nu})}$  and  $L_{\mu(\bar{\mu})}$  and mention the reason why *the SK assumption on the direction* does not hold.

## 2. Single Ring Events among Fully Contained Events which are Produced by Quasi Elastic Scattering.

### 2.1. Differential cross section of quasi elastic scattering and spreads of the scattering angles

As stated in Introduction, the finding of observation of the maximum oscillation in the  $L/E$  analysis is the ultimate verification of the finding of neutrino oscillation by Super-Kamiokande. For the examination of the Super-Kamiokande's assertion, we analyze  $L/E$  distribution of the single ring events among *Fully Contained Events*.

In order to examine the validity of *the SK assumption on the direction*, we consider the single

ring events due to the following quasi elastic scattering(QEL):

$$\begin{aligned}\nu_e + n &\longrightarrow p + e^- \\ \nu_\mu + n &\longrightarrow p + \mu^- \\ \bar{\nu}_e + p &\longrightarrow n + e^+ \\ \bar{\nu}_\mu + p &\longrightarrow n + \mu^+, \end{aligned} \quad (2)$$

The differential cross section for QEL is given as follows [11].

$$\begin{aligned} \frac{d\sigma_{\ell(\bar{\ell})}(E_{\nu(\bar{\nu})})}{dQ^2} = \\ \frac{G_F^2 \cos^2\theta_C}{8\pi E_{\nu(\bar{\nu})}^2} \left\{ A(Q^2) \pm B(Q^2) \left[ \frac{s-u}{M^2} \right] \right. \\ \left. + C(Q^2) \left[ \frac{s-u}{M^2} \right]^2 \right\}, \end{aligned} \quad (3)$$

where

$$\begin{aligned} A(Q^2) &= \frac{Q^2}{4} \left[ f_1^2 \left( \frac{Q^2}{M^2} - 4 \right) + f_1 f_2 \frac{4Q^2}{M^2} \right. \\ &\quad \left. + f_2^2 \left( \frac{Q^2}{M^2} - \frac{Q^4}{4M^4} \right) + g_1^2 \left( 4 + \frac{Q^2}{M^2} \right) \right], \\ B(Q^2) &= (f_1 + f_2) g_1 Q^2, \\ C(Q^2) &= \frac{M^2}{4} \left( f_1^2 + f_2^2 \frac{Q^2}{4M^2} + g_1^2 \right). \end{aligned}$$

The signs  $+$  and  $-$  refer to  $\nu_{\mu(e)}$  and  $\bar{\nu}_{\mu(e)}$  for charged current (c.c.) interactions, respectively. The  $Q^2$  denotes the four momentum transfer between the incident neutrino and the charged lepton. Details of other symbols are given in [11].

The relation among  $Q^2$ ,  $E_{\nu(\bar{\nu})}$ , energy of the incident neutrino,  $E_\ell$ , energy of the emitted charged lepton (muon or electron or their anti-particles) and  $\theta_s$ , scattering angle of the emitted lepton, is given as

$$Q^2 = 2E_{\nu(\bar{\nu})}E_\ell(1 - \cos\theta_s). \quad (4)$$

Also, energy of the emitted lepton is given by

$$E_\ell = E_{\nu(\bar{\nu})} - \frac{Q^2}{2M}. \quad (5)$$

Now, let us examine magnitude of the scattering angle of the emitted lepton in a quantitative way,

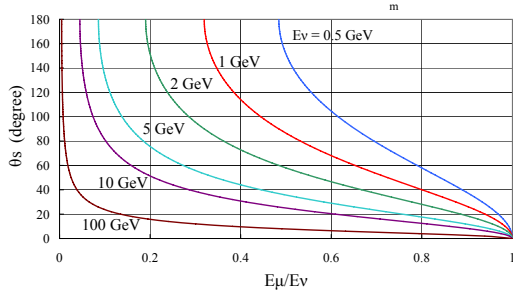


Figure 1: Relation between the energy of the muon and its scattering angle for different incident muon neutrino energies, 0.5, 1, 2, 5, 10 and 100 GeV.

as this plays a decisive role in determining the accuracy of direction of the incident neutrino, which is directly related to reliability of the zenith angle distribution of single ring muon (electron) events in the Super-Kamiokande Experiment. By using Eqs. (3) to (5), we obtain the distribution function for scattering angle of the emitted leptons and the related quantities by a Monte Carlo method. The procedure for determining scattering angle for a given energy of the incident neutrino is described in Appendix A. Figure 1 shows this relation for muon, from which we can easily understand that the scattering angle  $\theta_s$  of the emitted lepton (muon here) cannot be neglected. For a quantitative examination of the scattering angle, we construct the distribution function for  $\theta_s$  of the emitted lepton from Eqs. (3) to (5) by using the Monte Carlo method.

Figure 2 gives the distribution function for  $\theta_s$  of the muons produced in the muon neutrino interactions. It can be seen that the muons produced from lower energy neutrinos are scattered over wider angles and that a considerable part of them are scattered even in backward directions. Similar results are obtained for anti-muon neutrinos, electron neutrinos and anti-electron neutrinos.

Also, in a similar manner, we obtain not only the distribution function for the scattering angle of the charged leptons, but also their average values  $\langle \theta_s \rangle$  and their standard deviations  $\sigma_s$ . Table 1 shows them for muon neutrinos, anti-muon neutrinos, electron neutrinos and anti-electron neutrinos. From Table 1, it seems to be clear that the scattering angles could not be neglected, taking account of the fact that the frequency of the neutrino events with smaller energies is far larger than that of the neutrino events with larger energies due to high steep of the neutrino energy spectrum. However,

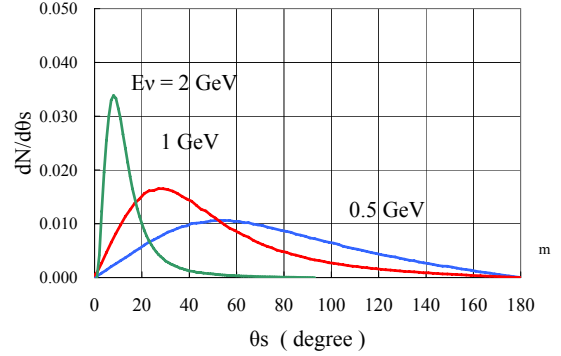


Figure 2: Distribution functions for the scattering angle of the muon for muon-neutrino with incident energies, 0.5 , 1.0 and 2 GeV. Each curve is obtained by the Monte Carlo method (one million sampling per each curve).

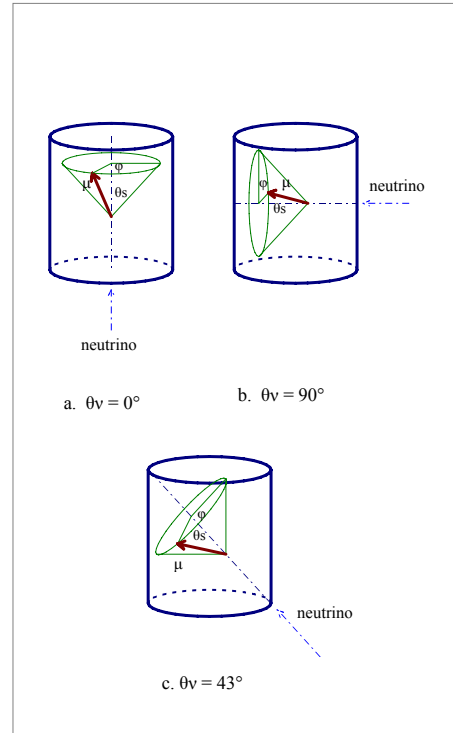


Figure 3: Schematic view of the zenith angles of the charged muons for different zenith angles of the incident neutrinos, focusing on their azimuthal angles.

Super-Kamiokande Collaboration assume that the direction of the neutrino is approximately the same as that of the emitted lepton even for the neutrino events with smaller energies, as cited in *the three passages* mentioned above [8], [9],[10]. However, this assumption has never been verified by Super-Kamiokande Collaboration.

### 2.2. Influence of the azimuthal angle in QEL over the zenith angle of single ring events

In the present subsection, we examine the effect of the azimuthal angles,  $\phi$ , of emitted leptons over their own zenith angles,  $\theta_{\mu(\bar{\mu})}$ , for given zenith angles of the incident neutrinos,  $\theta_{\nu(\bar{\nu})}$  in QEL, which was not be considered in the detector simulation carried by Super-Kamiokande Collaboration<sup>5</sup>. The influence of this effect over the zenith angle cannot be neglected particularly in horizontal-like neutrino events.

For three typical cases (vertical, horizontal and diagonal), Figure 3 gives a schematic representation of the relationship between,  $\theta_{\nu(\bar{\nu})}$ , the zenith angle of the incident neutrino, and  $(\theta_s, \phi)$ , a pair of scattering angle of the emitted muon and its azimuthal angle. Zenith angle of the emitted muon is derived from  $\theta_{\nu(\bar{\nu})}$  and  $(\theta_s, \phi)$  by (A.6) as shown in Appendix A.

From Figure 3-a, it can be seen that the zenith angle  $\theta_{\mu(\bar{\mu})}$  of the emitted lepton is not influenced by its  $\phi$  in the vertical incidence of the neutrinos ( $\theta_{\nu(\bar{\nu})} = 0^\circ$ ), as it must be. From Figure 3-b, however, it is obvious that the influence of  $\phi$  of the emitted leptons on their own zenith angle is the strongest in the case of horizontal incidence of the neutrino ( $\theta_{\nu(\bar{\nu})} = 90^\circ$ ). Namely, one half of the emitted leptons are recognized as upward going, while the other half is classified as downward going ones. The diagonal case ( $\theta_{\nu(\bar{\nu})} = 43^\circ$ ) is intermediate between the vertical and the horizontal. In the following, we examine the cases for vertical, horizontal and diagonal incidence of the neutrinos with different energies, say,  $E_{\nu(\bar{\nu})} = 0.5$  GeV,  $E_{\nu(\bar{\nu})} = 1$  GeV and  $E_{\nu(\bar{\nu})} = 5$  GeV, as the typical cases.

<sup>5</sup>Throughout this paper, we measure the zenith angles of the emitted leptons from the upward vertical direction of the incident neutrino. Consequently, notice that the sign of our direction is opposite to that of the Super-Kamiokande Experiment (our  $\cos \theta_{\nu(\bar{\nu})} = -\cos \theta_{\nu(\bar{\nu})}$  in SK)

### 2.3. Dependence of the spread of the zenith angle for emitted leptons on the energy of emitted leptons for different incident directions of the neutrinos with different energies

The detailed procedure for our Monte Carlo simulation is described in Appendix A. We give the scatter plots between the fractional energy of the emitted muons and their zenith angle for a definite zenith angle of the incident neutrino with different energies in Figures 4 to 6. In Figure 4, we give the scatter plots for vertically incident neutrinos with different energies 0.5, 1 and 5 GeV. In this case, the relations between the emitted energies of the muons and their zenith angles are unique, which comes from the definition of the zenith angle of the emitted lepton. However, the densities (frequencies of the event number) along each curves are different in position to position and depend on the energies of the incident neutrinos. Generally speaking, densities along the curves become greater toward  $\cos \theta_{\mu(\bar{\mu})} = 1$ . In this case,  $\cos \theta_{\mu(\bar{\mu})}$  is never influenced by the azimuthal angle in the scattering by the definition<sup>6</sup>.

On the contrast, it is shown in Figure 5 that the horizontally incident neutrinos give the widest zenith angle distribution for the emitted muons with the same fractional energies due to the effect of the azimuthal angles. The lower the energies of the incident neutrinos are, the wider the spreads of the scattering angles of emitted muons  $\theta_\mu$  become, which leads to wider zenith angle distributions for the emitted muons. As easily understood from Figure 6, the diagonally incident neutrinos give the intermediate zenith angle distributions for the emitted muons between those for vertically incident neutrinos and those for horizontally incident neutrinos.

It should be noticed from the figures that the influence of the azimuthal angle in QEL over the cosines of the zenith angle for the incident neutrino becomes effective in the more inclined neutrino, even if the scattering angle due to QEL is not so large. The effect of the azimuthal angle in QEL is not taken into account in the Monte Carlo simulation by Super-Kamiokande Collaboration, because their Monte Carlo simulation is a detector simulation and this effect cannot be taken into account by their nature.

<sup>6</sup> The zenith angles of the particles concerned are measured from the vertical upward direction.

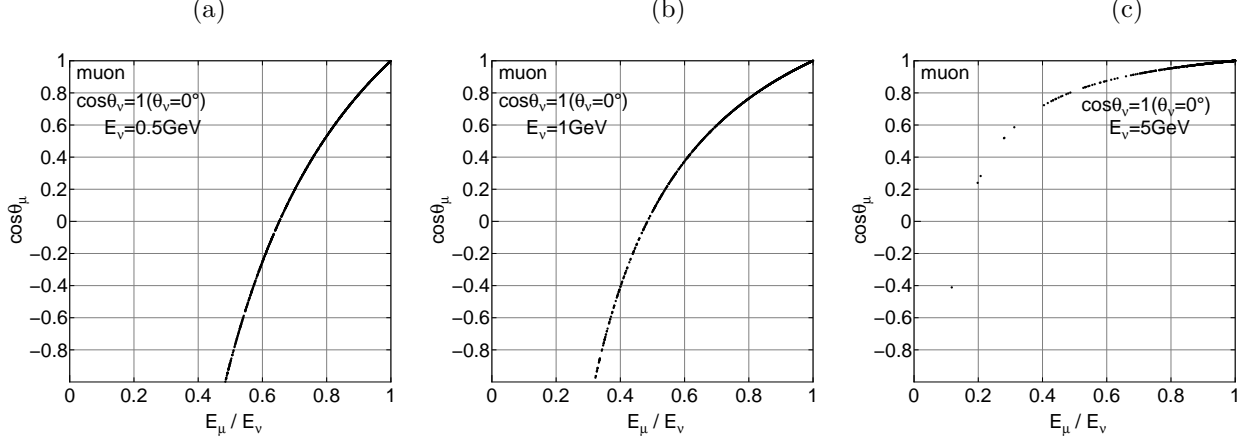


Figure 4: The scatter plots between the fractional energies of the produced muons and their zenith angles for vertically incident muon neutrinos with 0.5 GeV, 1 GeV and 5 GeV, respectively. The sampling number is 1000 for each case.

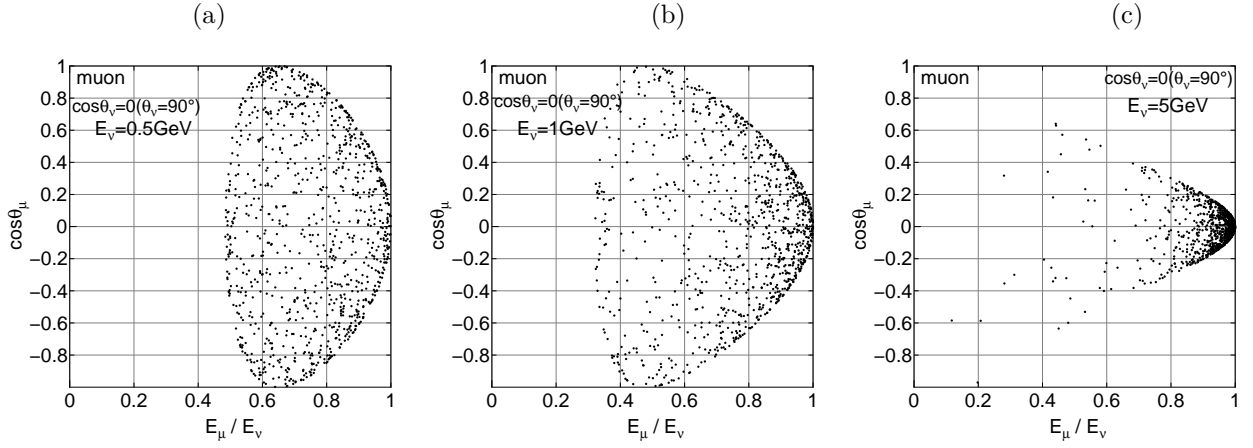


Figure 5: The scatter plots between the fractional energies of the produced muons and their zenith angles for horizontally incident muon neutrinos with 0.5 GeV, 1 GeV and 5 GeV, respectively. The sampling number is 1000 for each case.

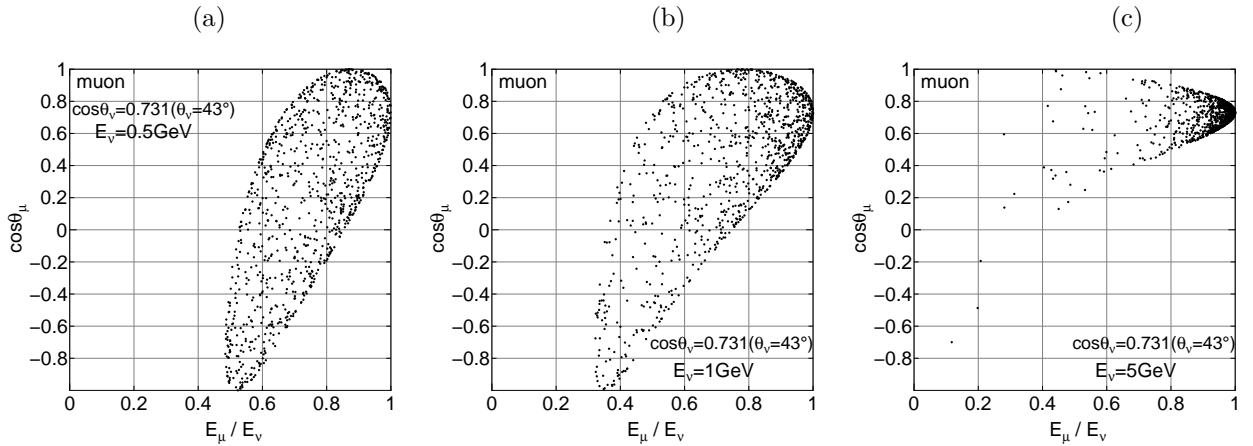


Figure 6: The scatter plots between the fractional energies of the produced muons and their zenith angles for diagonally incident muon neutrinos with 0.5 GeV, 1 GeV and 5 GeV, respectively. The sampling number is 1000 for each case.

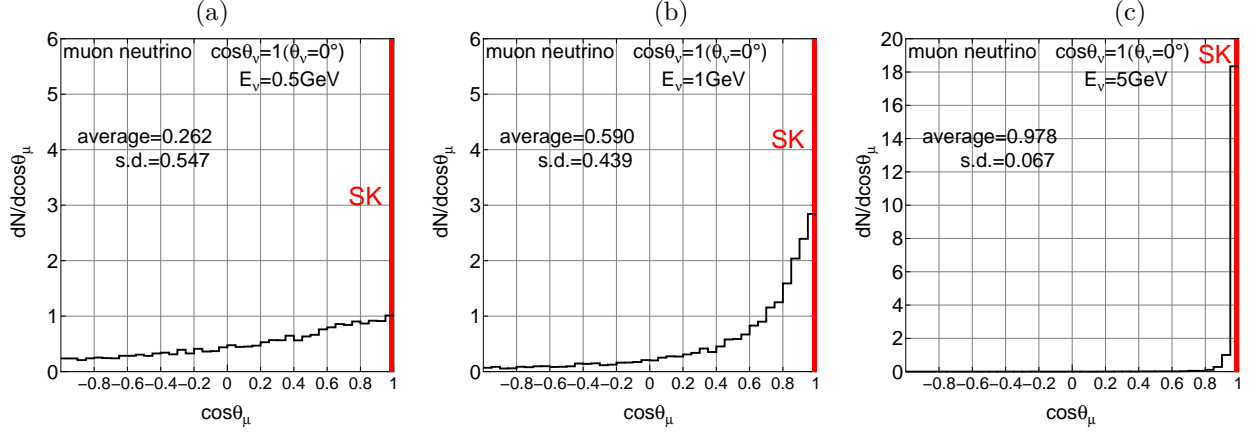


Figure 7: Zenith angle distribution of the muon for the vertically incident muon neutrino with 0.5 GeV, 1 GeV and 5 GeV, respectively. The sampling number is 10000 for each case. SK stands for the corresponding ones under *the SK assumption on the direction*.

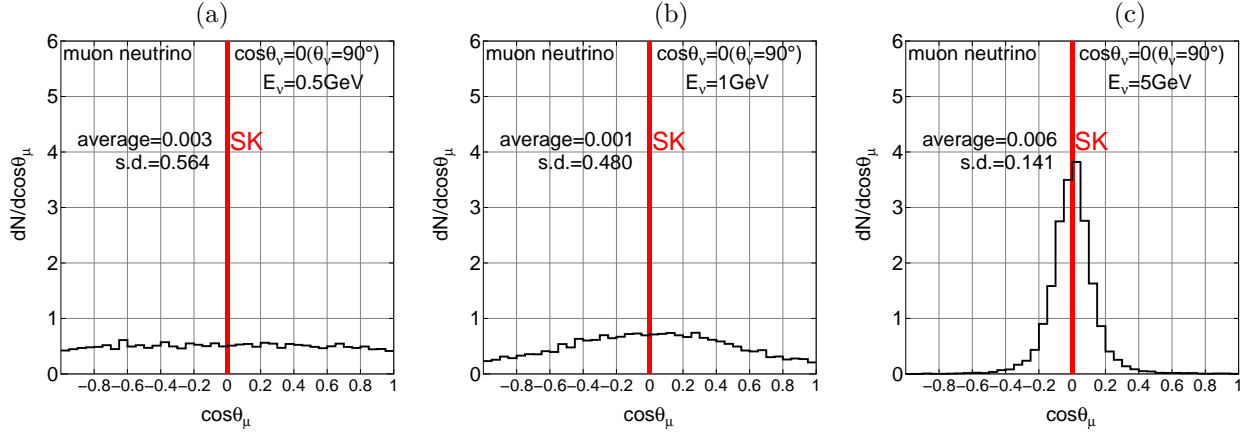


Figure 8: Zenith angle distribution of the muon for the horizontally incident muon neutrino with 0.5 GeV, 1 GeV and 5 GeV, respectively. The sampling number is 10000 for each case. SK stands for the corresponding ones under *the SK assumption on the direction*.

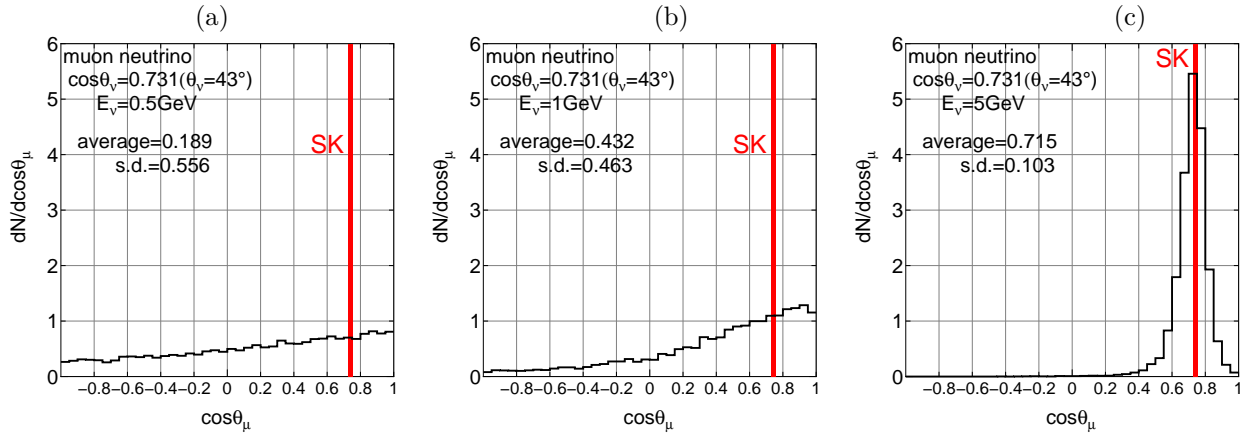


Figure 9: Zenith angle distribution of the muon for the diagonally incident muon neutrino with 0.5 GeV, 1 GeV and 5 GeV, respectively. The sampling number is 10000 for each case. SK stands for the corresponding ones under *the SK assumption on the direction*.



Table 1: The average values  $\langle \theta_s \rangle$  for scattering angle of the emitted charged leptons and their standard deviations  $\sigma_s$  for various primary neutrino energies  $E_{\nu(\bar{\nu})}$ .

$E_{\nu(\bar{\nu})}$ (GeV)	angle (degree)	$\nu_{\mu(\bar{\mu})}$	$\bar{\nu}_{\mu(\bar{\mu})}$	$\nu_e$	$\bar{\nu}_e$
0.2	$\langle \theta_s \rangle$	89.86	67.29	89.74	67.47
	$\sigma_s$	38.63	36.39	38.65	36.45
0.5	$\langle \theta_s \rangle$	72.17	50.71	72.12	50.78
	$\sigma_s$	37.08	32.79	37.08	32.82
1	$\langle \theta_s \rangle$	48.44	36.00	48.42	36.01
	$\sigma_s$	32.07	27.05	32.06	27.05
2	$\langle \theta_s \rangle$	25.84	20.20	25.84	20.20
	$\sigma_s$	21.40	17.04	21.40	17.04
5	$\langle \theta_s \rangle$	8.84	7.87	8.84	7.87
	$\sigma_s$	8.01	7.33	8.01	7.33
10	$\langle \theta_s \rangle$	4.14	3.82	4.14	3.82
	$\sigma_s$	3.71	3.22	3.71	3.22
100	$\langle \theta_s \rangle$	0.38	0.39	0.38	0.39
	$\sigma_s$	0.23	0.24	0.23	0.24

#### 2.4. Zenith angle distribution of the emitted leptons for different incident directions of the neutrinos with different energies

In Figures 7 to 9, we express Figures 4 to 6 in a different way. We sum up muon events with different emitted energies for given zenith angles. As the result of it, we obtain frequency distribution of the neutrino events as a function of  $\cos\theta_\mu$  for different incident directions and different incident energies of neutrinos.

In Figures 7(a) to 7(c), we give the zenith angle distributions of the emitted muons for the case of vertically incident neutrinos with different energies, say,  $E_\nu = 0.5, 1$  and  $5$  GeV.

Comparing the case for  $0.5$  GeV with that for  $5$  GeV, we understand the big contrast between them as for the zenith angle distribution. The scattering angle of the emitted muon for  $5$  GeV neutrino is relatively small (See, Table 1), so that the emitted muons keep roughly the same direction as their original neutrinos. In this case, the effect of their azimuthal angle on the zenith angle is also smaller. However, in the case for  $0.5$  GeV which is the dominant energy for single ring muon events in the Super-Kamiokande, there is even a possibility for the emitted muon to be emitted in the backward direction due to the larger angle scattering, the effect of which is enhanced by their azimuthal

angle.

The most frequent occurrence in the backward direction of the emitted muon appears in the horizontally incident neutrino as shown in Figs. 8(a) to 8(c). In this case, the zenith angle distribution of the emitted muon should be symmetrical with regard to the horizontal direction. Comparing the case for  $5$  GeV with those both for  $\sim 0.5$  GeV and for  $\sim 1$  GeV, even  $1$  GeV incident neutrinos lose almost the original sense of the incidence if we measure it by the zenith angle of the emitted muon. Figures 9(a) to 9(c) for the diagonally incident neutrinos tell us that the situation for diagonal case lies between the case for the vertically incident neutrinos and that for horizontally incident ones. SK in the figures denotes *the SK assumption on the direction* of incident neutrinos. From the Figures 7(a) to 9(c), it is clear that the scattering angles of emitted muons influence their zenith angles, which is enhanced by their azimuthal angles, particularly for more inclined directions of the incident neutrinos.

#### 2.5. Zenith Angle Distribution of Fully Contained Events for a Given Zenith Angle of the Incident Neutrino, Taking Their Energy Spectrum into Account

In the previous sections, we discuss the relation between the zenith angle distribution of the inci-

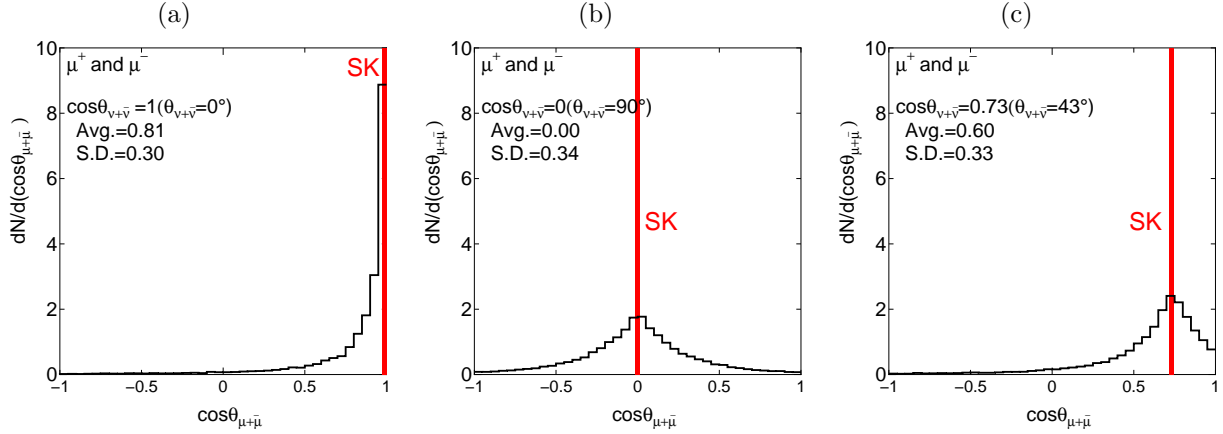


Figure 10: Zenith angle distribution of  $\mu^-$  and  $\mu^+$  for  $\nu$  and  $\bar{\nu}$  for the incident neutrinos with the vertical, horizontal and diagonal directions, respectively (see Figure 3). The overall neutrino spectra at Kamioka site are taken into account. The sampling number is 10000 for each case. SK stand for the corresponding ones under the SK assumption on the direction.

dent neutrino with a definite energy and that of the emitted muons produced by the neutrino for the different incident direction. In order to apply our inspection around the uncertainty of the SK assumption on the direction for Fully Contained Events to the real situation, we must consider the effect of the energy spectrum of the incident neutrino. The Monte Carlo simulation procedures for this purpose are given in Appendix B.

In Fig. 10, we give the zenith angle distributions of the sum of  $\mu^+(\bar{\mu})$  and  $\mu^-$  for a given zenith angle of  $\bar{\nu}_{\mu}$  and  $\nu_{\mu}$ , taking into account different primary neutrino energy spectra for different directions at Kamioka site. SK in the figures denotes the SK assumption on the direction. From the figures, it is clear that the SK assumption on the direction does not hold. Namely, we can conclude that the scattering angle of the emitted muons accompanied by their azimuthal angles influence their zenith angle distribution for all directions.

### 3. Super-Kamiokande Assumption on the Direction and the Real Relation between $\cos\theta_{\nu(\bar{\nu})}$ and $\cos\theta_{\mu(\bar{\mu})}$ and the corresponding relation between $L_{\nu(\bar{\nu})}$ and $L_{\mu(\bar{\mu})}$

#### 3.1. Correlation between $\cos\theta_{\nu}$ and $\cos\theta_{\mu}$

Now, we extend the results for the definite zenith angle obtained in the previous sections to the case in which we consider the zenith angle distribution of the incident neutrinos totally.

Here, we examine the real correlation between  $\cos\theta_{\nu}$  and  $\cos\theta_{\mu}$ , by performing the exact Monte Carlo simulation.

The detail for the simulation procedure is given in Appendix C.

In order to obtain the zenith angle distribution of the emitted leptons for that of the incident neutrinos, we divide the range of cosine of the zenith angle of the incident neutrino into twenty regular intervals from  $\cos\theta_{\nu} = 0$  to  $\cos\theta_{\nu} = 1$ . For the given interval of  $\cos\theta_{\nu}$ , we carry out the exact Monte Carlo simulation, and obtain the cosine of the zenith angle of the emitted leptons.

Thus, for each interval of  $\cos\theta_{\nu}$ , we obtain the corresponding zenith angle distribution of the emitted leptons. Then, we sum up these corresponding ones over all zenith angles of the incident neutrinos and we finally obtain the relation between the zenith angle distribution for the incident neutrinos and that for the emitted leptons.

In a similar manner, we could obtain between  $\cos\theta_{\bar{\nu}}$  and  $\cos\theta_{\bar{\mu}}$  for anti-neutrinos. The situation for anti-neutrinos is essentially the same as that for neutrinos.

Super-Kamiokande Collaboration have made the comprehensive analysis of the all possible types of the neutrino events on the neutrino oscillations [7] for 1489.2 live days. We restrict our analysis to the neutrino events due to QEL to make uncertainties around the interpretation small as possible.

In Figure 11, we give the correlation diagram between  $\cos\theta_{\nu}$  and  $\cos\theta_{\mu}$  for 1489.2 live days without oscillation. Here, we classify the correlations with regard to different incident neutrino energies. It is clear from Figure 11 that the neutrino events for  $E_{\nu} > 5$  GeV exist on the line

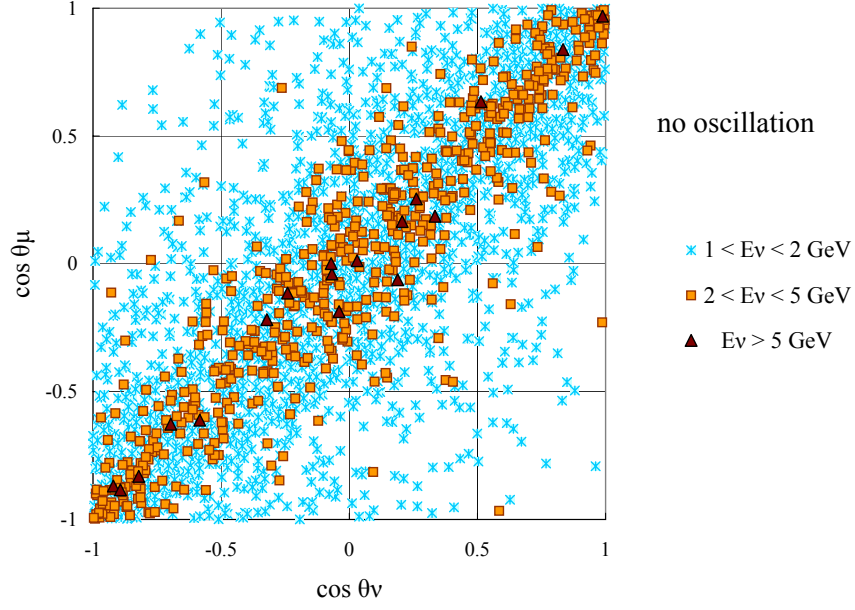


Figure 11: Correlation diagram between  $\cos \theta_\nu$  and  $\cos \theta_\mu$  for null oscillation for different neutrino energy regions.

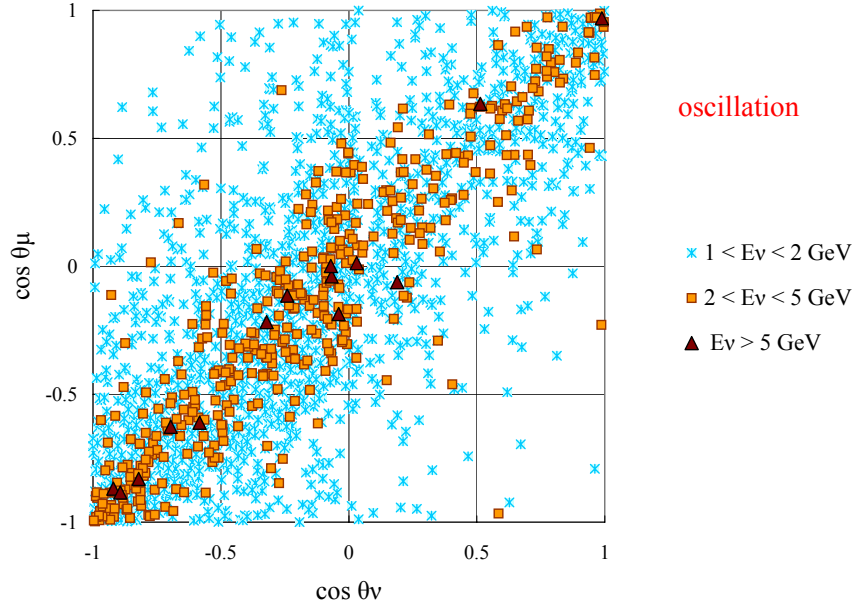


Figure 12: Correlation diagram between  $\cos \theta_\nu$  and  $\cos \theta_\mu$  for oscillation for different neutrino energy regions.

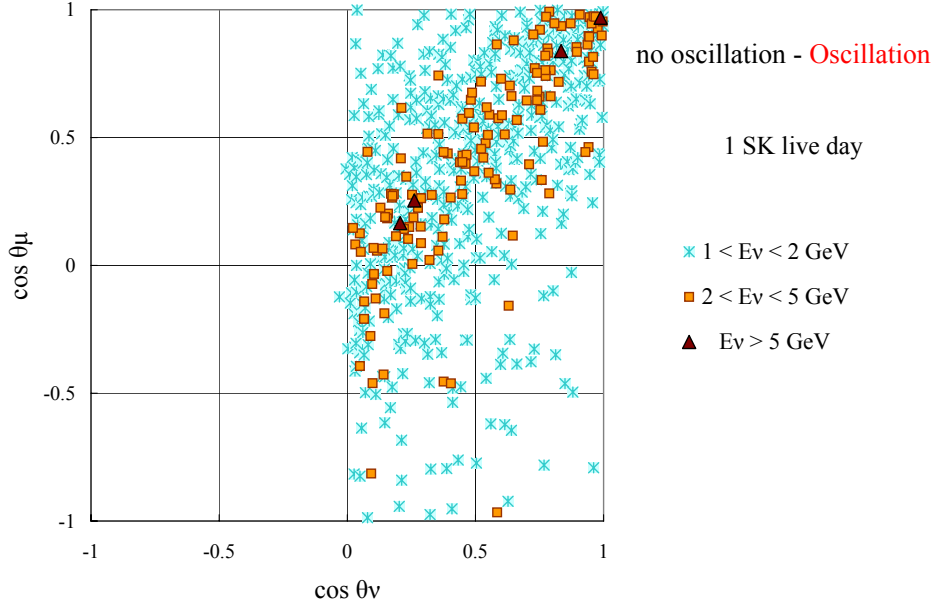


Figure 13: Correlation diagram between  $\cos \theta_\nu$  and  $\cos \theta_\mu$  for the events which exist in null oscillation but disappear due to oscillation for different neutrino energy regions.

$\cos \theta_\nu = \cos \theta_\mu$  roughly, and the neutrino events for  $2 < E_\nu < 5$  GeV exist along the line  $\cos \theta_\nu = \cos \theta_\mu$  with explicit deviations. Namely, in higher neutrino energy regions, *the SK assumption on the direction* roughly holds. However, if we pay our attention to lower energies, for  $1 < E_\nu < 2$  GeV, we easily understand that their distribution deviates largely from the line  $\cos \theta_\nu = \cos \theta_\mu$ , showing clearly that *the SK assumption on the direction* does not hold.

Consequently, in sum, we can conclude that *the SK assumption on the direction* does not hold as a whole, taking account of the fact that lower energy neutrino events occupy the majority in the detector due to the strong steepness of the incident neutrino energy spectra. In Figure 12, we give the correlation diagram between  $\cos \theta_\nu$  and  $\cos \theta_\mu$  for 1489.2 live days with oscillation. The Monte Carlo simulation with oscillation is carried out with the use of the rejection method with regard to the survival probability for a given flavor, which is based on the results obtained by the Monte Carlo simulation without oscillation<sup>7</sup>.

The regions in the correlation diagrams in Fig-

<sup>7</sup> We can carry out the Monte Carlo simulation for the case with oscillation quite independently from that without oscillation, of course. However, the adoption of the rejection

ures 11 and 12 are divided into the four parts, each of which have their own physical meaning. The neutrino events in the first sector where  $\cos \theta_\nu > 0$  and  $\cos \theta_\mu > 0$  consist of the upward neutrinos and upward muons. Namely, this denotes such a situation that the upward neutrinos emit the muons in forward directions. The neutrino events in the second sector where  $\cos \theta_\nu < 0$  and  $\cos \theta_\mu > 0$  consist of the downward neutrinos and the upward muons. Namely, this denotes such a situation that downward neutrinos emit the muons in backward directions. The neutrino events in the third sector where  $\cos \theta_\nu < 0$  and  $\cos \theta_\mu < 0$  consist of the downward neutrino and the downward muons. Namely, this denotes such a situation that the downward neutrinos emit the muons in forward directions. The neutrino events in the fourth sector where  $\cos \theta_\nu > 0$  and  $\cos \theta_\mu < 0$  consist of the upward neutrinos and the downward muons. Namely, this denotes such a situation that the upward neutrinos emit the muons in backward directions.

Here, the downward muons are generated in three different manners. In the first case, they are generated by downward neutrinos. In the second case,

method make clearer the effect of the survival probability for the given flavor.

they are generated by upward neutrinos, owing to the backscattering in QEL. And in the third case, they are generated by either upward or downward horizontal-like neutrinos. In the last case, either upward or downward horizontal-like neutrinos may produce downward muons due to the accidental azimuthal effect in QEL.(see, Figure 3b). The upward muons are generated in three different manners similarly by both upward and downward neutrinos.

It is clear from Figure 11 that events in the first sector are symmetrical to those in the third sector, while events in the second sector are symmetrical to those in the forth sector. As we expect the same incident neutrino fluxes in both downward and upward directions in the case of no oscillation, these symmetries are should be hold, as it must be. On the other hand, it is clear from Figure 12 that symmetries found in Figure 11 are lost any more. This comes from that the upward neutrino flux is smaller than that of the downward neutrino in the case of the existence of oscillation.

In Figure 13, we give the events which exist in the case of no oscillation but disappear due to oscillation obtained through the Monte Carlo operational procedure by the survival probability of a given flavor ( Eq.(1) ). It is simply obtained by the subtraction of the events in Figure 12 from the events in Figure 11.

We can obtain several interesting results from Figure 13. The first is that we do not find disappeared (rejected) events in the downward neutrino events due to oscillation. This reason is very simple and clear. It is due to the extreme low probability of oscillation for downward neutrino with the statistics considered on the occasion of the selection of the specified neutrino oscillation parameters by Super-Kamiokande Collaboration, say  $\Delta m^2 = 2.4 \times 10^{-3} \text{eV}^2$ . When we adopt this value one order of magnitude larger than SK's,  $\Delta m^2 = 2.4 \times 10^{-2} \text{eV}^2$ , we surely expect the events concerned for the downward neutrinos. Furthermore, when we adopt this value one order of magnitude smaller than SK's,  $\Delta m^2 = 2.4 \times 10^{-4} \text{eV}^2$ , then we surely expect different distribution of the events concerned even for the upward neutrinos, keeping the absence of the events concerned for downward neutrinos. The second is that almost disappeared (rejected) events belong to the first sector (upward neutrinos  $\rightarrow$  upward muons). The third is that we cannot neglect disappeared (rejected) neutrino events which belong to the forth sector (upward neutrino  $\rightarrow$  downward muon).

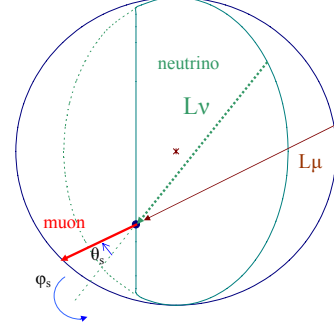


Figure 14: Schematic view of relations among  $L_\nu$ ,  $L_\mu$ ,  $\theta_s$  and  $\phi_s$ .

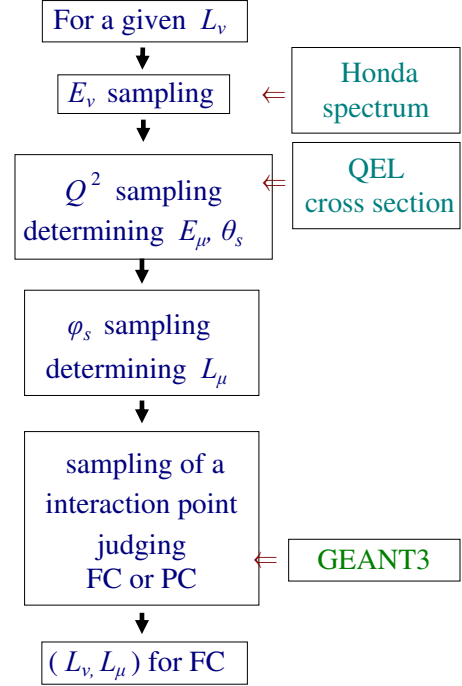


Figure 15: The procedure for our numerical experiment for obtaining  $L_\mu$  from a given  $L_\nu$ .

### 3.2. Correlation between $L_\nu$ and $L_\mu$

In the previous subsection, we verify that *the SK assumption on the direction* does not hold from the examination on the correlatin between  $\cos\theta_\nu$  and  $\cos\theta_\mu$ .

This SK assumption on  $\cos\theta_\nu$  and  $\cos\theta_\mu$  is logically equivalent to the statement that  $L_\nu$  is approximately the same as  $L_\mu$  in  $L/E$  analysis, where  $L_\nu$  denotes the distance on the incident neutrino from the interaction point of the neutrino events to the intersection of the Earth surface toward its arriving direction and  $L_\mu$  denotes the corresponding distance on the emitted muon. Consequently, if our indication on the invalidity of *the SK assumption on the direction* on  $\cos\theta_\nu$  and  $\cos\theta_\mu$  is correct, the same conclusion should be drawn from the correlation between  $L_\nu$  and  $L_\mu$ . In the present subsection, we directly examine the validity of the implicit SK assumption that  $L_\nu$  is approximated by  $L_\mu$ , taking consideration of the neutrino energy spectrum at the Super-Kamiokande site. The relation between  $L_\nu$  and  $L_\mu$  is given in Figure 14. Also, we show the procedure to determine  $L_\mu$  for a given  $L_\nu$  in Figure 15.

$L_\nu$  and  $L_\mu$  are functions of the direction cosine of the incident neutrino,  $\cos\theta_\nu$ , and that of emitted muon,  $\cos\theta_\mu$ , respectively and they are given as,

$$L_\nu = R_g \times (r_{SK} \cos\theta_\nu + \sqrt{r_{SK}^2 \cos^2\theta_\nu + 1 - r_{SK}^2}) \quad (6-1)$$

$$L_\mu = R_g \times (r_{SK} \cos\theta_\mu + \sqrt{r_{SK}^2 \cos^2\theta_\mu + 1 - r_{SK}^2}) \quad (6-2)$$

where  $R_g$  is the radius of the Earth and  $r_{SK} = 1 - D_{SK}/R_g$ , with the depth,  $D_{SK}$ , of the Super-Kamiokande Experiment detector from the surface of the Earth. It should be noticed that the  $L_\nu$  and  $L_\mu$  are regulated by both the energy spectrum of the incident neutrino and the production spectrum of the muon (QEL in the present case). Consequently, their mutual relation is influenced by either the absence of oscillation or the presence of oscillation which depend on the combination of oscillation parameters.

In Figures 16 and 17, we give the correlation diagram between  $L_\nu$  and  $L_\mu$  for single ring muon events among *Fully Contained Events* for the 1489.2 live days in both the absence and the present of neutrino oscillation which corresponds to the actual Super-Kamiokande Experiment[7]. In Figure 18, we give the subtraction of Figure 17 from Figure 16 which corresponds to Figure 13, exactly. In Figures 16 to 18, blue points denote neutrino events

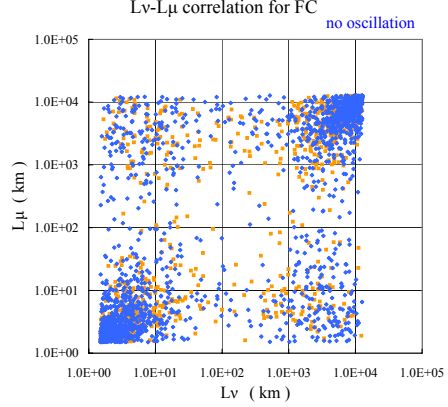


Figure 16: Correlation diagram for  $L_\nu$  and  $L_\mu$  without oscillation for 1489.2 live days. The blue points and orange points denote neutrino events and anti-neutrino events, respectively.

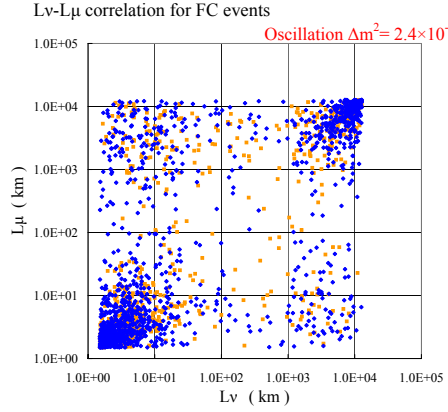


Figure 17: Correlation diagram for  $L_\nu$  and  $L_\mu$  with oscillation for 1489.2 live days. The blue points and orange points denote neutrino events and anti-neutrino events, respectively.

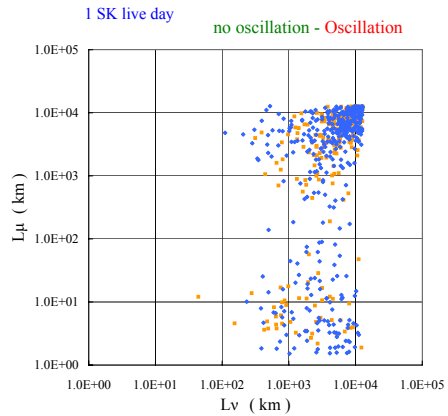


Figure 18: Correlation diagram for  $L_\nu$  and  $L_\mu$  for the events exist in null oscillation but disappear due to oscillation for 1489.2 live days. The blue points and orange points denote neutrino events and anti-neutrino events, respectively.

while orange points denote anti-neutrino events. Throughout all correlation diagrams in the present paper, blue points and orange ones have the same meaning as shown in these Figures. The aggregates of the (anti-) neutrino events which correspond to definite combinations of  $L_\nu$  and  $L_\mu$  are essentially classified into four groups in the following, and they correspond to four sections with regards to  $\cos\theta_\nu$  and  $\cos\theta_\mu$  as shown in Figures 11 to 13:

Group A is defined as the aggregate for neutrino events in which both  $L_\nu$  and  $L_\mu$  are rather small. It denotes that the downward neutrinos produce the downward muons with smaller scattering angles. In this case, energies of the produced muons are near those of the incident neutrinos due to smaller scattering angles.

Group B is defined as the aggregate for neutrino events in which both  $L_\nu$  and  $L_\mu$  are rather large. It denotes that the upward neutrinos produce upward muons with smaller scattering angles. In this case, the energy relation between incident neutrinos and produced muons is essentially the same as in Group A in Figure 16, because the flux of the upward neutrino events is symmetrical to that of downward neutrino events in the absence of neutrino oscillation.

Group C is defined as the aggregate for neutrino events in which  $L_\nu$  are rather small and  $L_\mu$  are rather large. It denotes that the downward neutrinos produce the upward muons by the possible effect resulting from both backscattering and azimuthal angle in QEL. In this case, energies of the produced muons are smaller than those of the incident neutrinos due to larger scattering angles.

Group D is defined as the aggregate for the neutrino events in which  $L_\nu$  are rather large and  $L_\mu$  are rather small. It denotes that the upward neutrinos produce the downward muons. The energy relation between the incident neutrinos and the produced muons is essentially the same as in Group C in the absence of neutrino oscillation (Figure 16).

Summarized from the mentioned above, we can say that there exist the symmetries between Group A and Group B, and also between Group C and Group D, which reflect the symmetry between the upward neutrino flux and the downward neutrino one for null oscillation.

In Figure 17, we give the correlation between  $L_\nu$  and  $L_\mu$  under their neutrino oscillation parameters, say,  $\Delta m^2 = 2.4 \times 10^{-3} \text{eV}^2$  and  $\sin^2 2\theta = 1.0$  [7]. In the presence of neutrino oscillation, the property of the symmetry which holds in the absence

of neutrino oscillation (see ⟨Group A and Group B⟩ and/or ⟨Group C and Group D⟩ in Figure 16) is lost due to different incident neutrino fluxes in the upward direction and the downward one. If we compare Group A with Group B, the event number in Group B (upward  $\nu \rightarrow$  upward  $\mu$ ) is smaller than that in group A (downward  $\nu \rightarrow$  downward  $\mu$ ), which comes from smaller flux of the upward neutrinos. The similar relation between Group C (downward  $\nu \rightarrow$  upward  $\mu$ ) and Group D (upward  $\nu \rightarrow$  downward  $\mu$ ) holds in Figure 17.

Summarizing the characteristics among Groups A to D in the Figures 16 and 17, we can conclude that ⟨Group A and Group B⟩ and ⟨Group C and Group D⟩ are in symmetrical situations in the absence of neutrino oscillation, while such a symmetrical situation is lost in the presence of neutrino oscillation. Also, it is clear from Figures 16 and 17 that  $L_\nu \approx L_\mu$ , namely *the SK assumption on the direction*, does not hold both in the absence of neutrino oscillation and in the presence of neutrino oscillation even if statistically.

Here, it should be noticed that the approximation of  $L_\nu \approx L_\mu$  does not hold completely in the regions C and D. The event numbers in Group C and Group D could not be neglected among the total event number concerned. In these regions, neutrino events consist of those with backscattering and/or neutrino events in which the neutrino directions are horizontally downward (upward), but their produced muons turn upward (downward) resulting from the effect of azimuthal angles in QEL.

In the  $L/E$  analysis made by Super-Kamiokande Collaboration, the reconstruction of the direction of the incident neutrino from the direction of the emitted muon is very simple, say,

$$\cos \Theta_\nu^{rec} = \cos \Theta_\mu$$

and there is nothing more <sup>8</sup>.

Of course, as we pointed out already, Super-Kamiokande Collaboration know the existence of the non-negligible scattering angles of the neutrino interaction. However, they don't consider it in their analysis. The concept of the "reconstruction" by Super-Kamiokande Collaboration denote one-to-one correspondence between the direction of the incident neutrino and the direction of the emitted lepton. However, the correlations between them shown

<sup>8</sup> following " $\cos \Theta_\nu^{rec} = \cos \Theta_\mu$  (8.17)" , Ishitsuka states " $\cos \Theta_\nu^{rec}$  and  $\cos \Theta_\mu$  are cosines of the reconstructed zenith angles of neutrino and muon, respectively." (see text in page 3) .



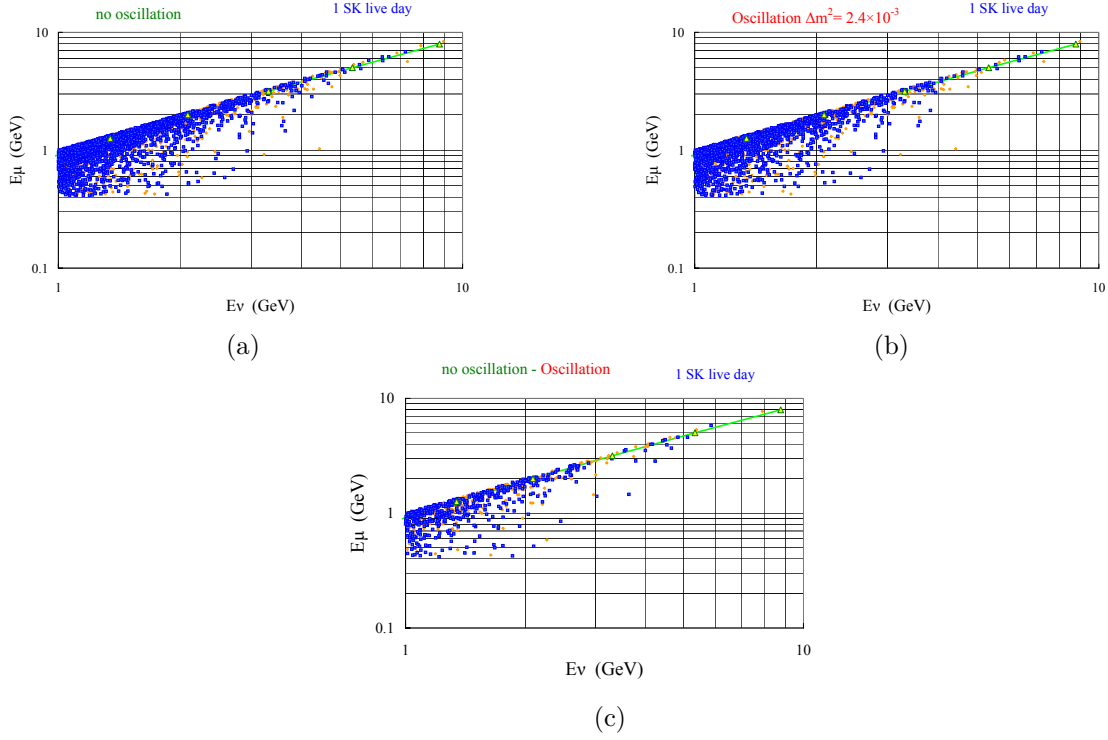


Figure 19: The correlation diagram between  $E_\nu$  and  $E_\mu$  (a) without oscillation, (b) with oscillation and (c) for the events which exist in null oscillation but disappear due to oscillation, for 1489.2 live days, respectively. The solid line denotes the polynomial expression by Super-Kamiokande Collaboration.

in Figures 16 and 17 make it impossible to reconstruct the direction of the incident neutrino from the direction of emitted muon, even if one takes the scattering angle into account, because these correlations are the reflection of the stochastic characters in the processes concerned which deny unique relation between them. Again, we should emphasize and confirm that "reconstruction of the direction of the incident neutrino" in  $L/E$  analysis of Super-Kamiokande Collaboration is just equal to the adoption of the direction of the emitted muon due to the neutrino interaction.

### 3.3. The correlation between $E_\nu$ and $E_\mu$

Here, we examine the transformation of  $E_\nu$  from  $E_\mu$  adopted by Super-Kamiokande Collaboration. The validity problem is closely related to *the SK assumption on the direction* through the survival probability for a given flavor.

Super-Kamiokande Collaboration estimate  $E_\nu$  from  $E_\mu$ , the visible energy of the muon, from their Monte Carlo simulation, by the following equation[9] (see page 135 of the paper concerned) :

$$E_{\nu,SK} = E_\mu \times (a + b \times x + c \times x^2 + d \times x^3), \quad (7)$$

where  $x = \log_{10}(E_\mu)$ .

The idea that  $E_\nu$  can be approximated as the polynomial of  $E_\mu$  means that there should be the

unique relation between  $E_\nu$  and  $E_\mu$ . However, in the light of stochastic characters inherent in both the incident neutrino energy spectrum and the production spectrum of the muon, such a treatment is not suitable theoretically, which may kill a real correlation effect between the incident neutrino energy and the emitted muon energy. In Figures 19, we give the correlation between  $E_\nu$  and  $E_\mu$  together with that obtained from the polynomial expression by Super-Kamiokande Collaboration under their neutrino oscillation parameters and their incident neutrino energy spectrum[12]. It is clear from the figure that the part of the lower energy incident neutrino deviates largely from the approximated formula, which reflects explicitly the stochastic character of QEL.

In Figure 19 (b), we give the correlation between  $E_\nu$  and  $E_\mu$  with oscillation. Glancing at Figures 19 (a) and (b), we cannot recognize the difference between them, because too much events are marked on the figure for their discrimination. Then in Figure 19 (c), we give the subtraction between them, as shown in Figures 13 and 18. It is easily understood that there is the visible difference between them. Furthermore, it is easily understood from Figures 19 that the polynomial expression is not suitable for the description on the mutual relation between  $E_\nu$  and  $E_\mu$ , standing on the stochastic



point of view.

Finally, it is necessary to mention the qualitative difference between the correlation between  $L_\nu$  and  $L_\mu$ , and the correlation between  $E_\nu$  and  $E_\mu$ , from the degree of their influence on *the SK assumption on the direction*. As clarified in Figures 16 and 17, it is impossible to approximate as  $L_\nu$  nearly equal to  $L_\mu$  ( $L_\nu \approx L_\mu$ ). On the other hand, the approximation of  $E_\nu$  by Eq.(7) does not introduce fatal error finally, although its treatment is not theoretically suitable in the sense of the lack of stochastic characters on the physical processes. Namely, what influences on the  $L/E$  analysis essentially is the correlation between  $L_\nu$  and  $L_\mu$ , but not the correlation between  $E_\nu$  and  $E_\mu$ .

#### 4. Conclusion

Since one cannot measure  $L_\nu$  and  $E_\nu$ , one is forced to utilize  $L_\mu$  and  $E_\mu$  in the  $L/E$  analysis in place of them. Then, Super-Kamiokande Collaboration assume that the direction of the incident neutrino is approximately the same as that of the emitted lepton, namely *the SK assumption on the direction* and  $E_\nu$  can be estimated from some polynomial formula of the variable  $E_\mu$  in their  $L/E$  analysis. However, it is clear from Figures 12 and 13, and/or Figures 16 and 17, that *the SK assumption on the direction* does not hold even approximately and the transformation of  $E_\mu$  into  $E_\nu$  is not uniquely.

Consequently, we can conclude that  $L_\nu$  cannot be reconstructed by  $L_\mu$ , but  $E_\nu$  can be done if we allow considerable uncertainties.

In the subsequent paper (Part 2), we apply the results from Figures 16 to 19 to  $L/E$  analysis and conclude that one cannot obtain the maximum oscillation in  $L/E$  analysis reported by Super-Kamiokande which shows strongly the oscillation pattern due to neutrino oscillation.

## APPENDICES

### Appendix A. Monte Carlo Procedure for the Decision of Emitted Energies of the Leptons and Their Direction Cosines

Here, we give the Monte Carlo Simulation procedure for obtaining the energy and its direction cosines,  $(l_r, m_r, n_r)$ , of the emitted lepton in QEL for a given energy and its direction cosines,  $(l, m, n)$ , of the incident neutrino.

The relation among  $Q^2$ ,  $E_{\nu(\bar{\nu})}$ , the energy of the incident neutrino,  $E_{\ell(\bar{\ell})}$ , the energy of the emitted lepton (muon or electron or their anti-particles) and  $\theta_s$ , the scattering angle of the emitted lepton, is given as

$$Q^2 = 2E_{\nu(\bar{\nu})}E_{\ell(\bar{\ell})}(1 - \cos\theta_s). \quad (\text{A.1})$$

Also, the energy of the emitted lepton is given by

$$E_{\ell(\bar{\ell})} = E_{\nu(\bar{\nu})} - \frac{Q^2}{2M}. \quad (\text{A.2})$$

#### Procedure 1

We decide  $Q^2$  from the probability function for the differential cross section with a given  $E_{\nu(\bar{\nu})}$  (Eq. (3) in the text) by using the uniform random number,  $\xi$ , between (0,1) in the following

$$\xi = \int_{Q_{\min}^2}^{Q^2} P_{\ell(\bar{\ell})}(E_{\nu(\bar{\nu})}, Q^2) dQ^2, \quad (\text{A.3})$$

where

$$P_{\ell(\bar{\ell})}(E_{\nu(\bar{\nu})}, Q^2) = \frac{d\sigma_{\ell(\bar{\ell})}(E_{\nu(\bar{\nu})}, Q^2)}{dQ^2} \bigg/ \int_{Q_{\min}^2}^{Q_{\max}^2} \frac{d\sigma_{\ell(\bar{\ell})}(E_{\nu(\bar{\nu})}, Q^2)}{dQ^2} dQ^2. \quad (\text{A.4})$$

From Eq. (A.1), we obtain  $Q^2$  in histograms together with the corresponding theoretical curve shown in Figure A.1. The agreement between the sampling data and the theoretical curve is excellent, which shows the validity of the utilized procedure in Eq. (A.3).

#### Procedure 2

We obtain  $E_{\ell(\bar{\ell})}$  from Eq. (A.2) for the given  $E_{\nu(\bar{\nu})}$

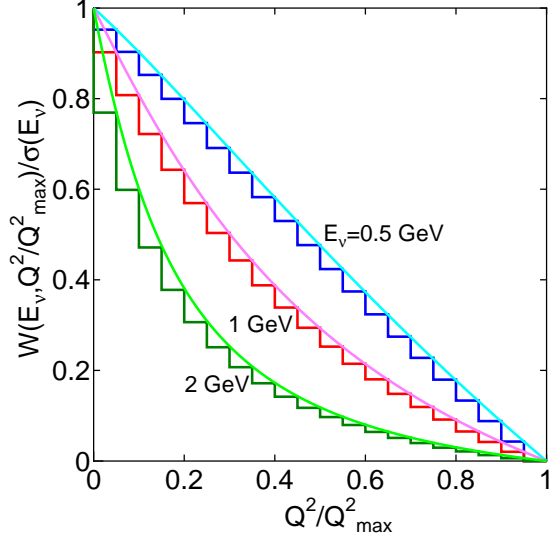


Figure A.1: The reappearance of the probability function for QEL cross section. Histograms are sampling results, while the curves concerned are theoretical ones for given incident energies.

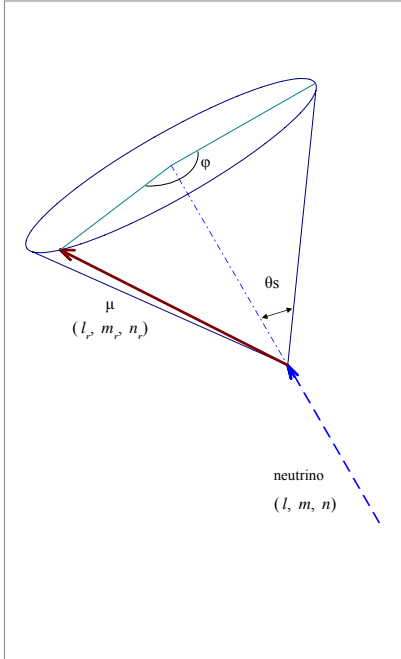


Figure A.2: The relation between the direction cosine of the incident neutrino and that of the emitted charged lepton.

and  $Q^2$  thus decided in Procedure 1.

### Procedure 3

We obtain  $\cos\theta_s$ , cosine of the the scattering angle of the emitted lepton, for  $E_{\ell(\bar{\ell})}$  thus decided in the Procedure 2 from Eq. (A.1) .

### Procedure 4

We decide  $\phi$ , the azimuthal angle of the scattered lepton, which is obtained from

$$\phi = 2\pi\xi. \quad (\text{A.5})$$

Here,  $\xi$  is a uniform random number between (0, 1).

As explained schematically in the text(see Figure 3 in the text), we must take account of the effect due to the azimuthal angle  $\phi$  in QEL to obtain the zenith angle distribution both for *Fully Contained Events* and *Partially Contained Events* correctly.

### Procedure 5

The relation between direction cosines of the incident neutrino,  $(\ell_{\nu(\bar{\nu})}, m_{\nu(\bar{\nu})}, n_{\nu(\bar{\nu})})$ , and those of the corresponding emitted lepton,  $(\ell_r, m_r, n_r)$ , for a certain  $\theta_s$  and  $\phi$  is given as

$$\begin{pmatrix} \ell_r \\ m_r \\ n_r \end{pmatrix} = \begin{pmatrix} \frac{\ell n}{\sqrt{\ell^2 + m^2}} & -\frac{m}{\sqrt{\ell^2 + m^2}} & \ell_{\nu(\bar{\nu})} \\ \frac{mn}{\sqrt{\ell^2 + m^2}} & \frac{\ell}{\sqrt{\ell^2 + m^2}} & m_{\nu(\bar{\nu})} \\ -\sqrt{\ell^2 + m^2} & 0 & n_{\nu(\bar{\nu})} \end{pmatrix} \times \begin{pmatrix} \sin\theta_s \cos\phi \\ \sin\theta_s \sin\phi \\ \cos\theta_s \end{pmatrix}, \quad (\text{A.6})$$

where  $n_{\nu(\bar{\nu})} = \cos\theta_{\nu(\bar{\nu})}$ , and  $n_r = \cos\theta_\ell$ . Here,  $\theta_\ell$  is the zenith angle of the emitted lepton.

The Monte Carlo procedure for the determination of  $\theta_\ell$  of the emitted lepton for the parent (anti-)neutrino with given  $\theta_{\nu(\bar{\nu})}$  and  $E_{\nu(\bar{\nu})}$  involves the following steps:

We obtain  $(\ell_r, m_r, n_r)$  by using Eq. (A.6). The  $n_r$  is the cosine of the zenith angle of the emitted lepton which should be contrasted with  $n_\nu$ , that of the incident neutrino.

Repeating the procedures 1 to 5 just mentioned

above, we obtain the zenith angle distribution of the emitted leptons for a given zenith angle of the incident neutrino with a definite energy.

In the SK analysis, instead of Eq. (A-6), they assume  $n_r = n_{\nu(\bar{\nu})}$  uniquely for  $E_{\mu(\bar{\mu})} \geq 400$  MeV.

## Appendix B. Monte Carlo Procedure to Obtain the Zenith Angle of the Emitted Lepton for a Given Zenith Angle of the Incident Neutrino

The present simulation procedure for a given zenith angle of the incident neutrino starts from the atmospheric neutrino spectrum at the opposite site of the Earth to the SK detector. We define,  $N_{\text{int,no-osc}}(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})})$ , the interaction neutrino spectrum at the depth  $t$  from the SK detector for the case no oscillation in the following way,

$$\begin{aligned} N_{\text{int,no-osc}}(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})}) = & N_{\text{sp}}(E_{\nu(\bar{\nu})}, \cos\theta_{\nu(\bar{\nu})}) \times \\ & \left(1 - \frac{dt}{\lambda_1(E_{\nu(\bar{\nu})}, t_1, \rho_1)}\right) \times \\ & \times \cdots \times \left(1 - \frac{dt}{\lambda_n(E_{\nu(\bar{\nu})}, t_n, \rho_n)}\right). \end{aligned} \quad (\text{B}\cdot 1)$$

Here,  $N_{\text{sp}}(E_{\nu(\bar{\nu})}, \cos\theta_{\nu(\bar{\nu})})$  is the atmospheric (anti-) neutrino spectrum for the zenith angle at the opposite surface of the Earth. And  $\lambda_i(E_{\nu(\bar{\nu})}, t_i, \rho_i)$  denotes the mean free path from QEL for the neutrino (anti neutrino) with the energy  $E_{\nu(\bar{\nu})}$  at the distance,  $t_i$ , from the opposite surface of the Earth, where  $\rho_i$  is there density.

In the presence of oscillation, neutrino energy spectrum corresponding to (B-1) is given as,

$$\begin{aligned} N_{\text{int,osc}}(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})}) &= N_{\text{int,no-osc}}(E_{\nu(\bar{\nu})}, \cos\theta_{\nu(\bar{\nu})}) \times P(\nu_\mu \rightarrow \nu_\mu) \end{aligned} \quad (\text{B}\cdot 2)$$

Here,  $P(\nu_\mu \rightarrow \nu_\mu)$  is the survival probability of a given flavor, such as  $\nu_\mu$ , and it is given by

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \cdot \sin^2(1.27 \Delta m^2 L_\nu / E_\nu), \quad (\text{B}\cdot 3)$$

where  $\sin^2 2\theta = 1.0$  and  $\Delta m^2 = 2.4 \times 10^{-3} \text{eV}^2$  obtained from Super-Kamiokande Collaboration[7].

The procedures of the Monte Carlo Simulation for the incident neutrino(anti neutrino) with a given energy,  $E_{\nu(\bar{\nu})}$ , whose incident direction is expressed by  $(l, m, n)$  are as follows.

### Procedure A

For the given zenith angle of the incident neutrino,  $\theta_{\nu(\bar{\nu})}$ , we formulate,  $N_{\text{pro}}(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})})dE_{\nu(\bar{\nu})}$ , the production function for the neutrino flux to produce leptons at the Kamioka site in the following

$$\begin{aligned} N_{\text{pro}}(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})})dE_{\nu(\bar{\nu})} &= \sigma_{\ell(\bar{\ell})}(E_{\nu(\bar{\nu})})N_{\text{int}}(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})})dE_{\nu(\bar{\nu})}, \end{aligned} \quad (\text{B}\cdot 4)$$

where

$$\sigma_{\ell(\bar{\ell})}(E_{\nu(\bar{\nu})}) = \int_{Q_{\text{min}}^2}^{Q_{\text{max}}^2} \frac{d\sigma_{\ell(\bar{\ell})}(E_{\nu(\bar{\nu})}, Q^2)}{dQ^2} dQ^2. \quad (\text{B}\cdot 5)$$

Each differential cross section above is given by Eq. (3) in the text. Here, we simply denote the interaction energy spectrum as  $N_{\text{int}}(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})})$ , irrespective of the absence or the presence of oscillation.

Utilizing,  $\xi$ , the uniform random number between (0,1), we determine  $E_{\nu(\bar{\nu})}$ , the energy of the incident neutrino in the following sampling procedure

$$\xi = \int_{E_{\nu(\bar{\nu}),\text{min}}}^{E_{\nu(\bar{\nu})}} P_d(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})})dE_{\nu(\bar{\nu})}, \quad (\text{B}\cdot 6)$$

where

$$\begin{aligned} P_d(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})})dE_{\nu(\bar{\nu})} &= \frac{N_{\text{pro}}(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})})dE_{\nu(\bar{\nu})}}{\int_{E_{\nu(\bar{\nu}),\text{min}}}^{E_{\nu(\bar{\nu}),\text{max}}} N_{\text{pro}}(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})})dE_{\nu(\bar{\nu})}}. \end{aligned} \quad (\text{B}\cdot 7)$$

In our Monte Carlo procedure, the reproduction of,  $P_d(E_{\nu(\bar{\nu})}, t, \cos\theta_{\nu(\bar{\nu})})dE_{\nu(\bar{\nu})}$ , the normalized differential neutrino interaction probability function, is confirmed in the same way as in Eq. (A-4).

### Procedure B

For the (anti-)neutrino concerned with the energy of  $E_{\nu(\bar{\nu})}$ , we sample  $Q^2$  utilizing  $\xi$ , the uniform random number between (0,1). The Procedure B is exactly the same as Procedure 1 in Appendix A.

### Procedure C

We decide,  $\theta_s$ , the scattering angle of the emitted lepton for given  $E_{\nu(\bar{\nu})}$  and  $Q^2$ . Procedure C is exactly the same as in the combination of Procedures 2 and 3 in Appendix A.

### Procedure D

We randomly sample the azimuthal angle of the charged lepton concerned. The Procedure D is exactly the same as in Procedure 4 in Appendix A.

### Procedure E

We decide the direction cosine of the charged lepton concerned. Procedure E is exactly the same as Procedure 5 in Appendix A.

We repeat Procedures A to E until we reach the desired trial number.

## Appendix C. Correlation between the Zenith Angles of the Incident Neutrinos and Those of the Emitted Leptons

### Procedure A

By using,  $N_{\text{pro}}(E_{\nu(\bar{\nu})}, t, \cos \theta_{\nu(\bar{\nu})}) dE_{\nu(\bar{\nu})}$ , which is defined in Eq. (B.4), we define the spectrum for  $\cos \theta_{\nu(\bar{\nu})}$  in the following.

$$I(\cos \theta_{\nu(\bar{\nu})}) d(\cos \theta_{\nu(\bar{\nu})}) = d(\cos \theta_{\nu(\bar{\nu})}) \times \int_{E_{\nu(\bar{\nu}), \min}}^{E_{\nu(\bar{\nu}), \max}} N_{\text{pro}}(E_{\nu(\bar{\nu})}, t, \cos \theta_{\nu(\bar{\nu})}) dE_{\nu(\bar{\nu})}. \quad (\text{C.1})$$

By using Eq. (C.2) and  $\xi$ , a sampled uniform random number between (0,1), then we can determine  $\cos \theta_{\nu(\bar{\nu})}$  from the following equation

$$\xi = \int_0^{\cos \theta_{\nu(\bar{\nu})}} P_n(\cos \theta_{\nu(\bar{\nu})}) d(\cos \theta_{\nu(\bar{\nu})}), \quad (\text{C.2})$$

where

$$P_n(\cos \theta_{\nu(\bar{\nu})}) = \frac{I(\cos \theta_{\nu(\bar{\nu})})}{\int_0^1 I(\cos \theta_{\nu(\bar{\nu})}) d(\cos \theta_{\nu(\bar{\nu})})}. \quad (\text{C.3})$$

### Procedure B

For the sampled  $d(\cos \theta_{\nu(\bar{\nu})})$  in Procedure A, we sample  $E_{\nu(\bar{\nu})}$  from Eq.(C.4) by using  $\xi$ , the uniform random number between (0,1)

$$\xi = \int_{E_{\nu(\bar{\nu}), \min}}^{E_{\nu(\bar{\nu})}} P_{\text{pro}}(E_{\nu(\bar{\nu})}, \cos \theta_{\nu(\bar{\nu})}) dE_{\nu(\bar{\nu})}, \quad (\text{C.4})$$

where

$$P_{\text{pro}}(E_{\nu(\bar{\nu})}, t, \cos \theta_{\nu(\bar{\nu})}) dE_{\nu(\bar{\nu})} = \frac{N_{\text{pro}}(E_{\nu(\bar{\nu})}, t, \cos \theta_{\nu(\bar{\nu})}) dE_{\nu(\bar{\nu})}}{\int_{E_{\nu(\bar{\nu}), \min}}^{E_{\nu(\bar{\nu}), \max}} N_{\text{pro}}(E_{\nu(\bar{\nu})}, t, \cos \theta_{\nu(\bar{\nu})}) dE_{\nu(\bar{\nu})}}. \quad (\text{C.5})$$

### Procedure C

For the sampled  $E_{\nu(\bar{\nu})}$  in Procedure B, we sample  $E_{\mu(\bar{\mu})}$  from Eqs. (A.2) and (A.3). For the sampled  $E_{\nu(\bar{\nu})}$  and  $E_{\mu(\bar{\mu})}$ , we determine  $\cos \theta_s$ , the scattering angle of the muon uniquely from Eq. (A.1).

### Procedure D

We determine,  $\phi$ , the azimuthal angle of the scattering lepton from Eq. (A.5) by using  $\xi$ , an uniform random number between (0,1).

### Procedure E

We obtain  $\cos \theta_{\mu(\bar{\mu})}$  from Eq. (A.6). As the result, we obtain a pair of  $(\cos \theta_{\nu(\bar{\nu})}, \cos \theta_{\mu(\bar{\mu})})$  through Procedures A to E. Repeating the Procedures A to E, we finally obtain the correlation between the zenith angle of the incident neutrino and that of the emitted muon.

## References

- [1] Hirata, KS *et al.*, Phys.Lett.**B205**(1988)416.  
Hirata, KS *et al.*, Phys.Lett.**B280**(1992)146.  
Casper, D *et al.*, Phys.Rev.Lett.**66**(1991)2561.  
Becker-Szendy, R *et al.*, Phys.Rev. D **46**(1992)3720.
- [2] Hatakeyama, S *et al.*, Phys.Rev.Lett.**81**(1998)2010.
- [3] Kajita, T, Neutrino 98, Takayama, Japan, June 4-9 19982.  
Fukuda, Y, Phys.Rev.Lett.**81**(1998)1562.
- [4] Mann, WA, Nucl.Phys.Proc.Supple Vol.**91**(2000)134.  
Ambrosio, Met *al.*, Phys.Lett.**B478**(2000)3.
- [5] K2K, Phys.Rev. D**74**(2006)72003.
- [6] Michael, DG *et al.*, Phys.Rev.Lett.**97**(2006) 191801.
- [7] Ashie,Y. *et al.*, Phys. Rev. D **71** (2005) 112005.
- [8] Kajita, T. and Totsuka, Y. Rev. Mod. Phys., **73** (2001)85. See p.101.
- [9] Ishitsuka, M, Ph.D thesis, University of Tokyo (2004).  
See p. 138.
- [10] Jung, CK, Kajita, T, Mann, T and McGrew, C, Anual.  
Rev. Mod. Sci. **vol.15** (2005) 431.
- [11] Renton, P., *Electro-weak Interaction*, Cambridge University Press (1990). See p. 405.
- [12] Honda, M., *et al.*, Phys. Rev. D **52** (1996) 4985.  
Honda, M., *et al.*, Phys. Rev. D **70** (2004)043008-1.
- [13] Ashie,Y *et al.*, Phys.Rev.Lett.**93** (2004)101801-1.